Kinematic Techniques for Missing Energy Events at Hadron Colliders

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Introduction

- Missing energy signals are important channels for new physics searches (e.g., SUSY-like theories containing a WIMP dark matter particle). They appear in Higgs search channels (with neutrinos).
- The full kinematics is difficult to reconstruct on an event-by-event basis with more than one missing particles in an event.
- Many kinematic variables and techniques have been developed to handle collider signals with missing energy.

Introduction

- Early kinematic variables are often heuristic and empirical (M_{eff}, H_T, ...). Recently, many variables based on more theoretically sound footings were discovered (M_{T2}, $\sqrt{\hat{s}_{min}}$,....).

- I will not go over all these variables or techniques, but just discuss the basic ideas behind them so that one can find a uniform understanding of them and look for new ones which are useful.

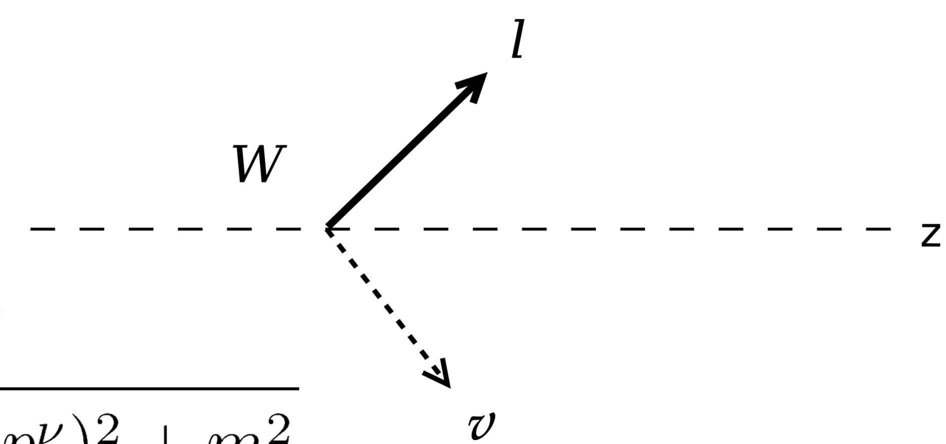
Kinematic boundaries

- A basic idea is to find the minimum (or maximum) mass or energy which is consistent with a given event for a hypothesized event topology, using all (or most relevant) kinematic constraints (mass-shell, missing transverse momentum).

- It has natural generalizations to higher-dim parameter space. One can find the region consistent with a given event. The true model parameters lie on the boundary (end point) of the intersected region of all signal events.

Examples

- Transverse mass M_T:



$$\alpha_\ell = (E_T^\ell, p_x^\ell, p_y^\ell), \quad \alpha_\nu = (E_T^\nu, p_x^\nu, p_y^\nu)$$

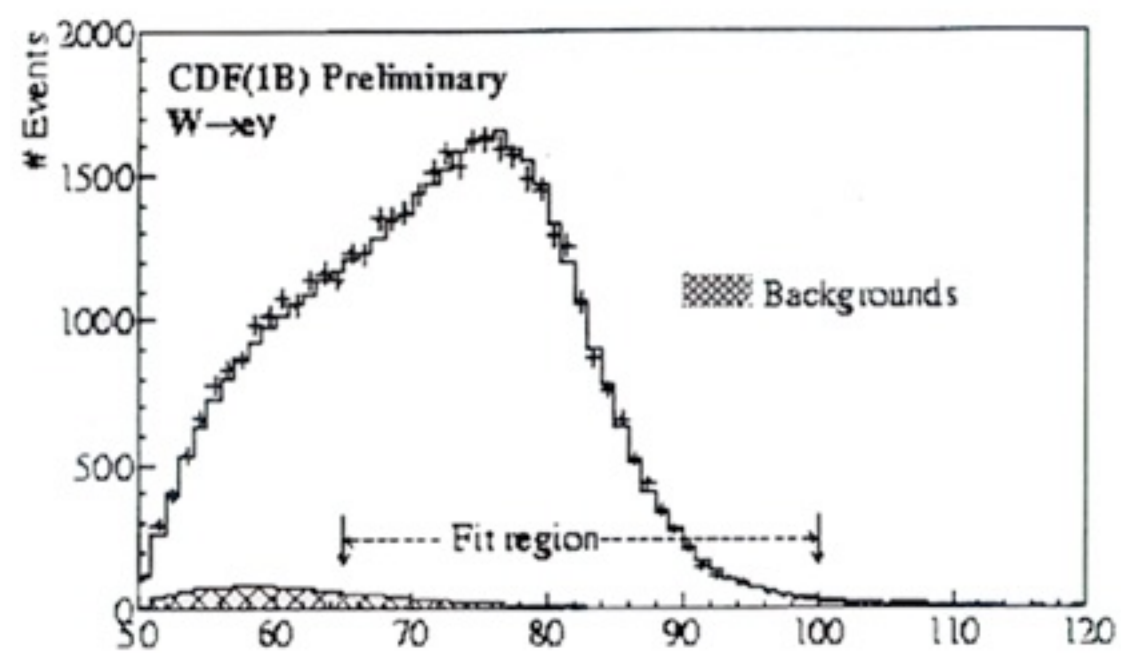
$$E_T^\ell = \sqrt{(p_x^\ell)^2 + (p_y^\ell)^2 + m_\ell^2}, \quad E_T^\nu = \sqrt{(p_x^\nu)^2 + (p_y^\nu)^2 + m_\nu^2}.$$

Transverse mass is defined by $M_T^2 = (\alpha_\ell + \alpha_\nu)^2$

For each event, M_T is the smallest mother particle (W) mass which can be consistent with that event, for a given invisible particle mass (m_ν). It occurs when l and ν have the same rapidity.

The end point of M_T distribution is the correct M_W.

W Transverse mass (CDF)

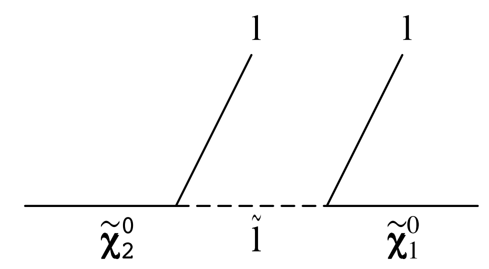


Examples

- Invariant mass of visible particles a decay chain

Hinchliffe et al, hep-ph/9610544, and many others

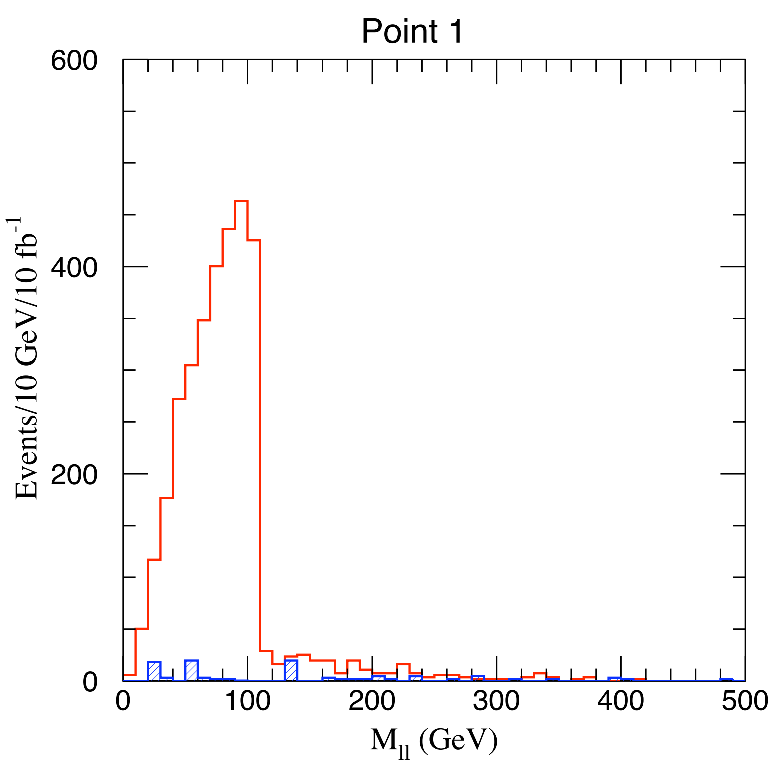
Example: the dilepton edge.



The invariant mass of the 2 visible particles can be viewed as a kinematic constraint on the combination of mass parameters:

$$\frac{(m^2_{\tilde{\chi}_2} - m^2_{\tilde{l}})(m^2_{\tilde{l}} - m^2_{\tilde{\chi}_1})}{m^2_{\tilde{l}}} \geq m^2_{ll}$$

Edge at $M_{ll} = \frac{\sqrt{(M^2_{\tilde{\chi}^0_2} - M^2_{\tilde{l}})(M^2_{\tilde{l}} - M^2_{\tilde{\chi}^0_1})}}{M_{\tilde{l}}}$



It defines a boundary between the allowed and the forbidden regions in the 3-dim mass space.

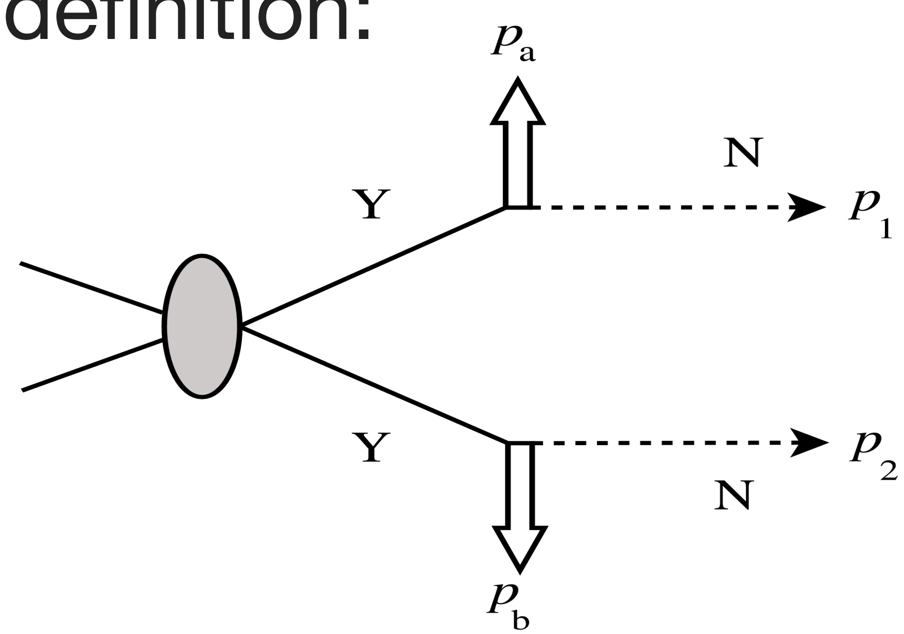
The end point of the invariant mass distribution provides one relation among 3 unknown masses.

F. E. Paige, hep-ph/9609373

Examples

- Stransverse mass M_{T2}: (Lester & Summers, hep-ph/9906349)

The original definition:



- Trial N mass, μ_N
- Consider all partitions of $\not{p}_T = \mathbf{p}_T^{(1)} + \mathbf{p}_T^{(2)}$.

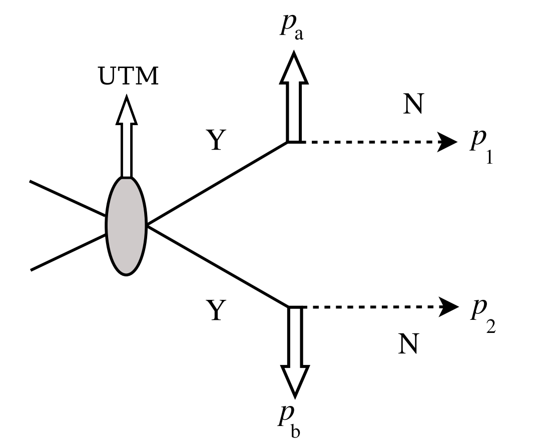
$$M_{T2}(\mu_N) \equiv \min_{\mathbf{p}_T^{(1)}+\mathbf{p}_T^{(2)}=\not{p}_T} \left[\max\{M_T(1,a;\mu_N), M_T(2,b;\mu_N)\}\right]$$

It's a function of the missing particle mass μ_N.

The end point of M_{T2} distribution gives the correct mother particle mass m_Y for the true μ_N.

Examples

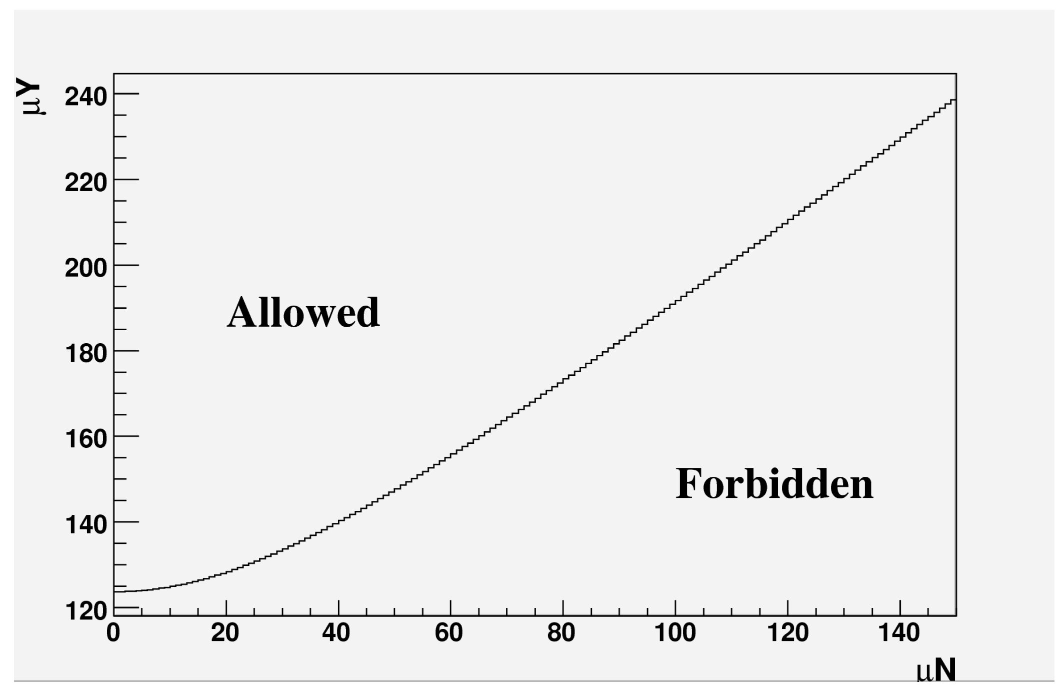
- M_{T2} can be understood as minimal kinematic constraints: (HC & Z. Han, arXiv:0810.5178)



$$p_1^2 = p_2^2 = \mu_N^2,$$
$$(p_1 + p_a)^2 = (p_2 + p_b)^2 = \mu_Y^2,$$
$$p_1^x + p_2^x = \not{p}^x,\ p_1^y + p_2^y = \not{p}^y,$$

  - Minimal kinematic constraints: mass shell constraints of the decaying mother particles and the missing particles + missing transverse momentum constraint.

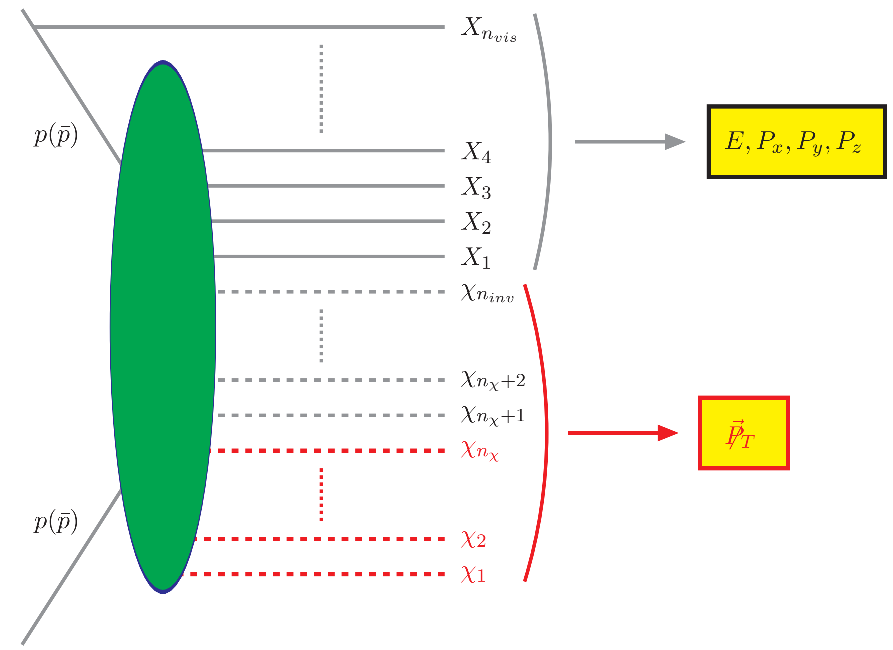
  - $M_{T2}(\mu_N)$ of a single event is the boundary of the allowed and the forbidden regions in the 2-dim mass space based on the minimal kinematic constraints of that event.



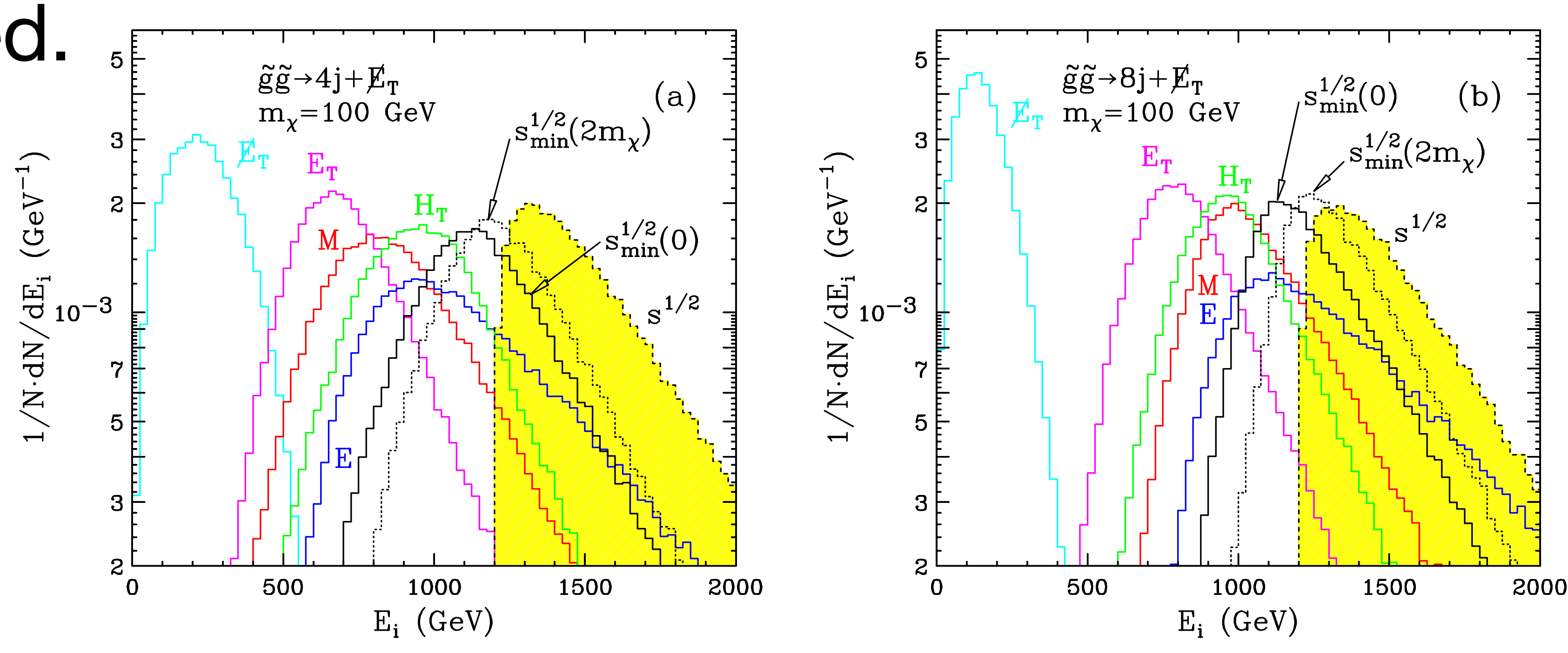
Examples

- Florida $\sqrt{\hat{s}_{min}}$: (Konar, P et al, arXiv:0812.1042,1006.0653)
  - Minimum center-of-mass parton-level energy consistent with total measured energy and visible momentum.

$$\hat{s}^{1/2}_{min}(M_{inv}) \equiv \sqrt{E^2 - P_z^2} + \sqrt{\not{E}_T^2 + M_{inv}^2}$$



  - A global and fully inclusive variable which can be used to measure the mass scale of the parent particles originally produced.

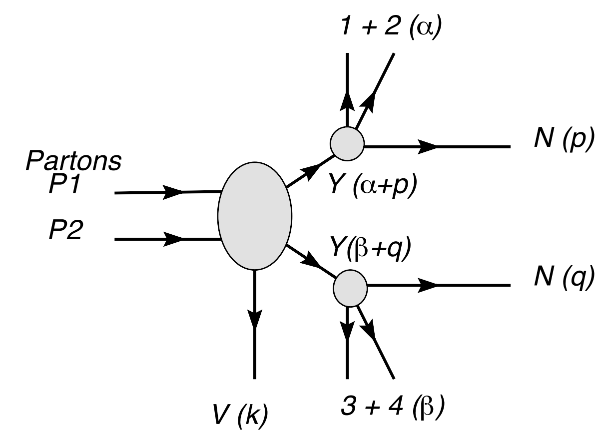


Applications

- Mass determinations in SUSY-like theories

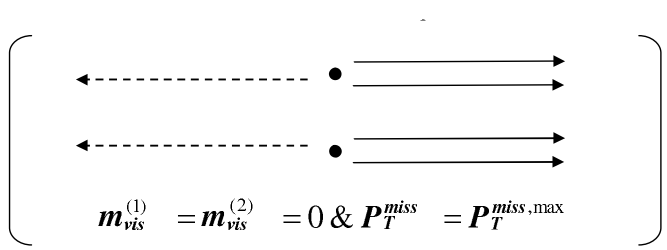
To determine a system of *N* unknown masses, we need at least *N* independent mass relations

$$\begin{aligned} \mathcal{O}_1(p_{i,\text{visible}}) &= f_1(m_1, m_2, \cdots), \\ \mathcal{O}_2(p_{i,\text{visible}}) &= f_2(m_1, m_2, \cdots), \\ &\vdots \end{aligned}$$



Example: M_{T2} kink (Cho, et al, arXiv:0709.0288, 0711.4526; Barr, et al 0711.4008)

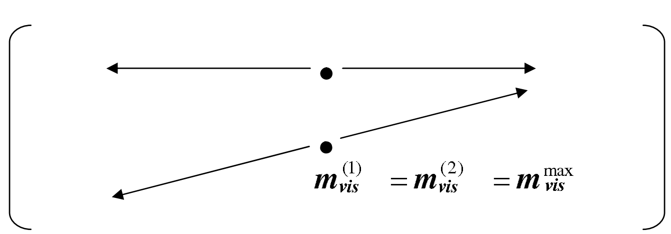
Blue:



$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} + \sqrt{\left(\frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}}\right)^2 + m_\chi^2}$$

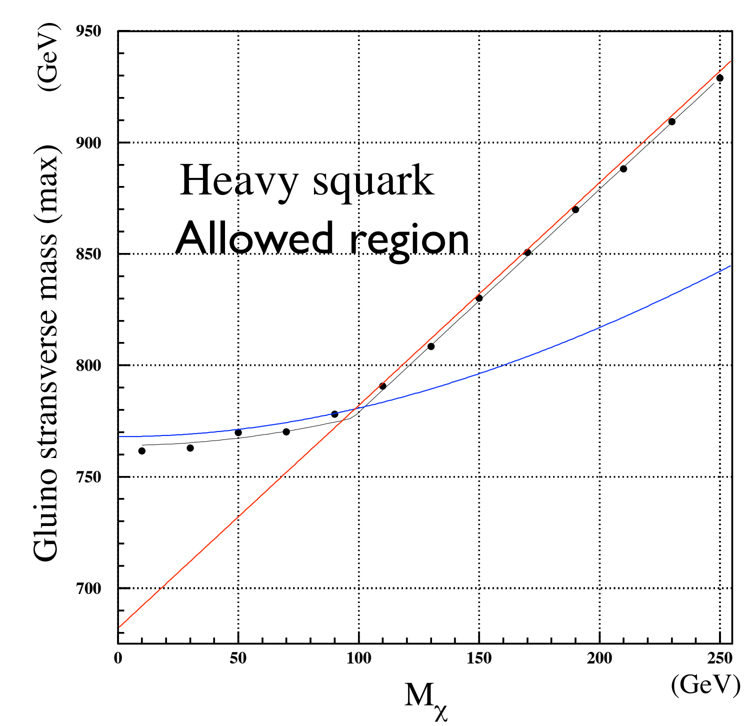
if $m_\chi < m_{\tilde{\chi}_1^0}$

Red:



$$m_{T2}^{\max}(m_\chi) = \left(m_{\tilde{g}} - m_{\tilde{\chi}_1^0}\right) + m_\chi$$

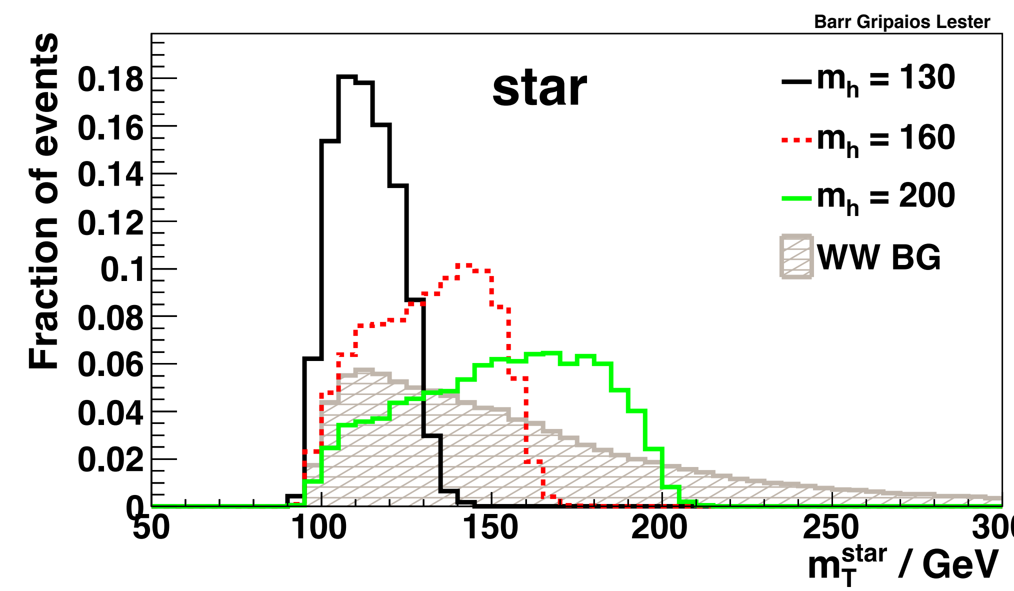
if $m_\chi > m_{\tilde{\chi}_1^0}$



Applications

- Higgs searches
  - $h \to WW^{(*)} \to \ell\nu\ell\nu$ (Barr, et al, arXiv:1108.3468)

    $m_{\mathrm{T}}^{\star}$: Smallest Higgs mass which can be consistent with a given event by requiring one *W* on-shell. The end point is the true Higgs mass.



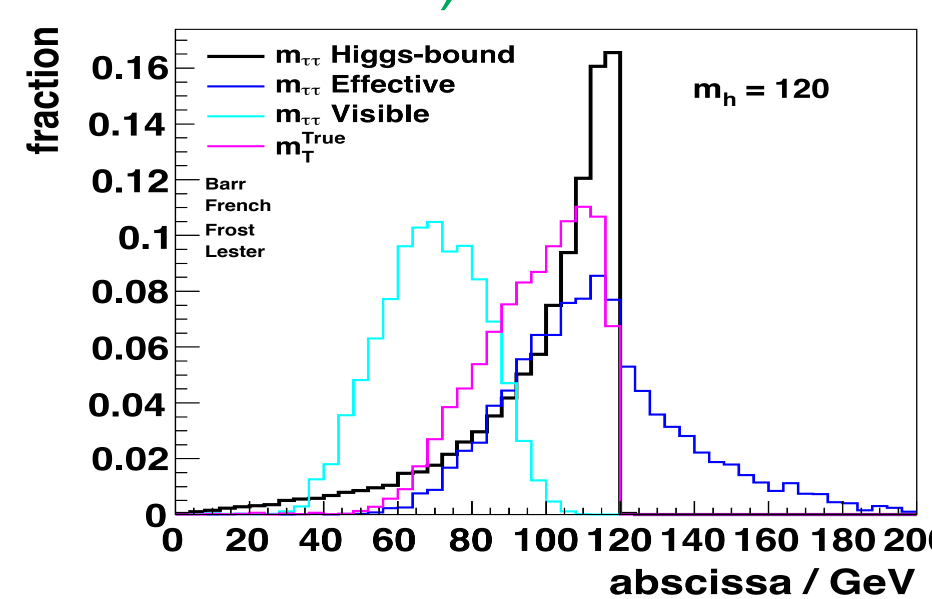
  - $h \to \tau\tau \to \ell\nu\nu\ell\nu\nu$ (Barr, et al, arXiv:1106.2322)

$$m_{\tau\tau}^{\mathrm{Higgs-bound}} = \min_{\{Q_1^\mu, Q_2^\mu | \aleph\}} \sqrt{H^\mu H_\mu}$$

$$H^\mu = P_1^\mu + Q_1^\mu + P_2^\mu + Q_2^\mu$$

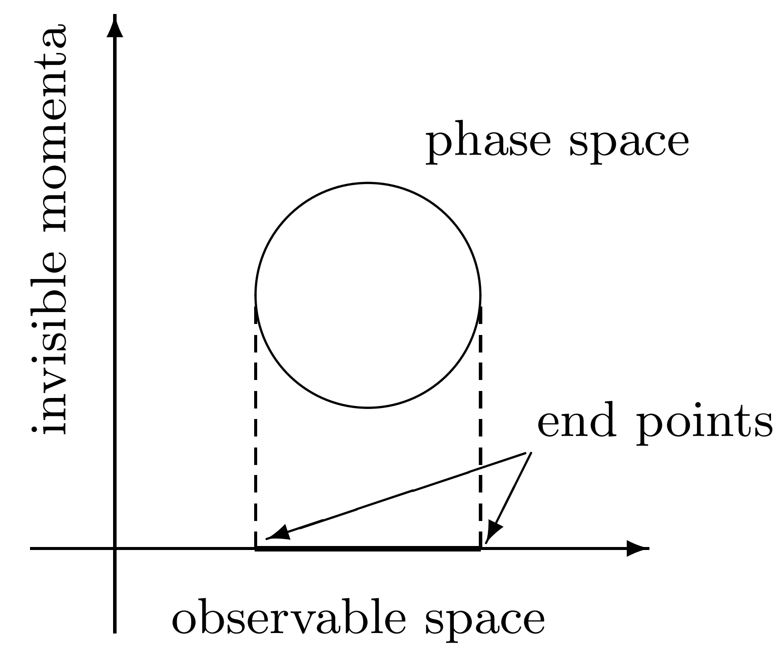
$P_{1,2}^\mu$:measured lepton momenta

$Q_{1,2}^\mu$:hypothesized neutrino momenta



Degenerate constraint equations

- The constraint equations restrict the allowed phase space in some hyper-surface in the space of all visible and invisible momenta.
- Boundaries or end points arise when it's projected to the subspace of visible momenta.



- The boundaries are often where signal events accumulate and hence useful for searches. The events at or near the boundaries are also most important for mass measurements.

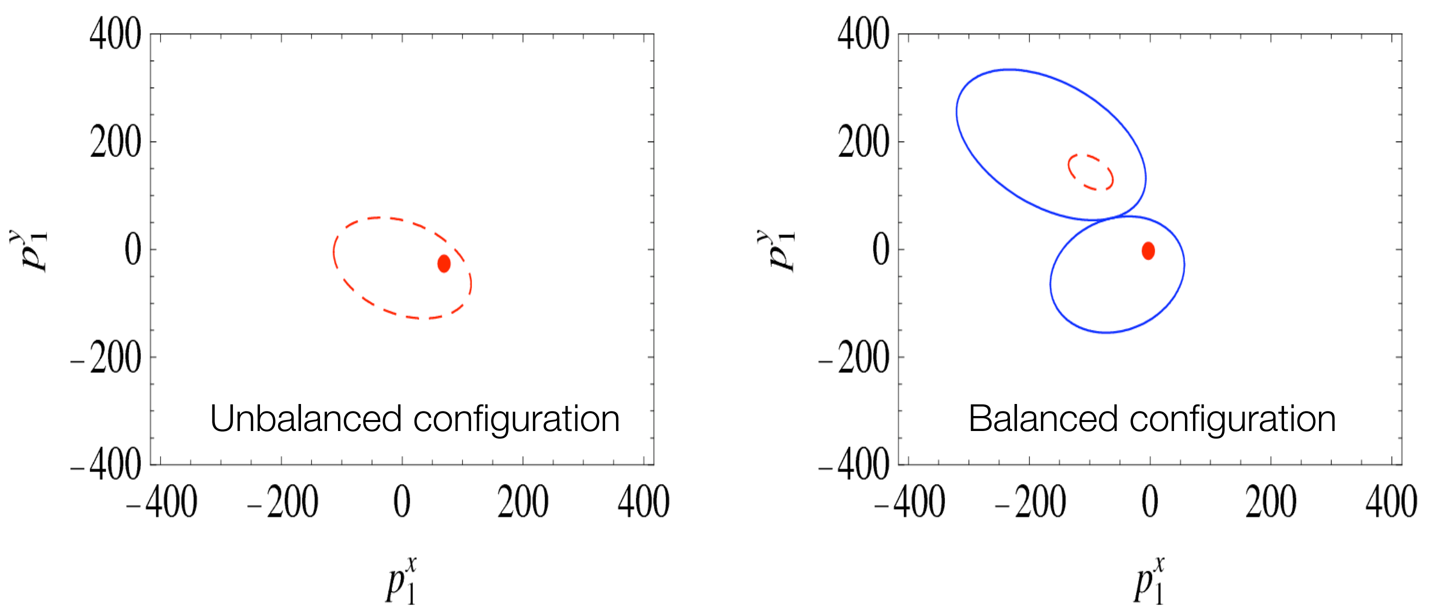
Degenerate constraint equations

- Alternative point of view: boundary events make the (tangent of) constraint equations degenerate. Existence of solutions implies certain relations among coefficients which depends on the masses.

- End points occur because some constraint equations are quadratic (mass-shell). It has been shown that the invariant mass and M_{T2} end points can be understood in this way. (Kim, I.-W. arXiv:0910.1149)

Ex: M_{T2} calculation

(HC & Z. Han, arXiv:0810.5178)



Degenerate constraint equations

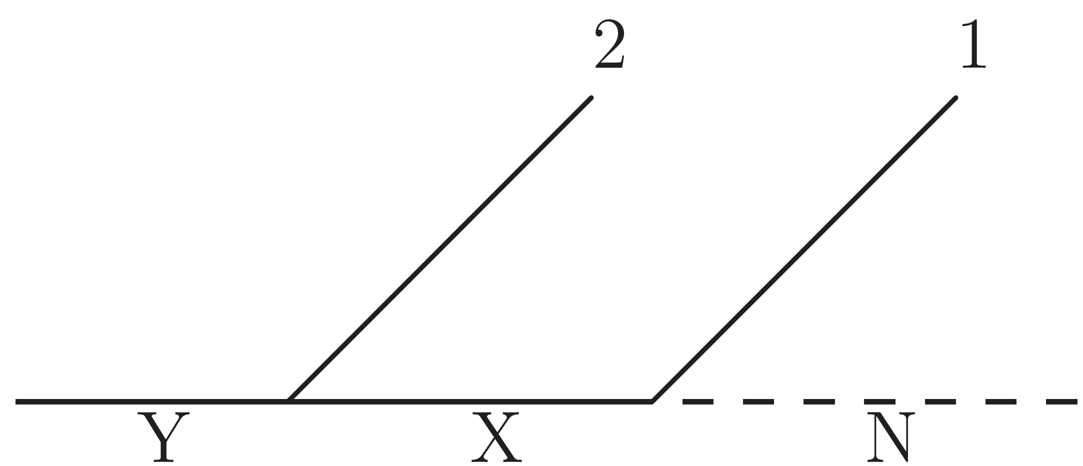
- However, not all degeneracies occur at the end points. There may be more special events to use beyond those at the end points.

- We will study an example in which the degeneracy occurs between linear equations. It allows us to do the mass measurement for a topology which was not possible before.

A single short decay chain

HC and Jiayin Gu, to appear

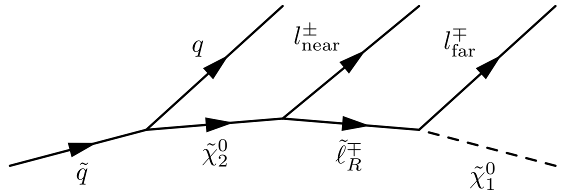
- We consider the mass measurement for the following decay chain at a hadron collider.



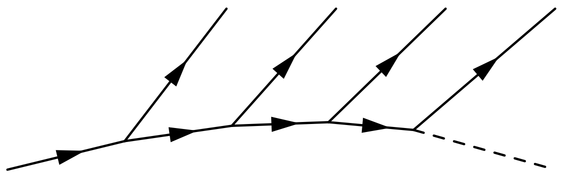
There is at least another missing particle in the event, so the transverse momentum of *N* is unknown.

Motivations

- Appears in many models with WIMP dark matter.
- Most recent studies focus on symmetric decay chains, but there may be much more asymmetric-chain events than symmetric ones.
- Mass measurements were only done for long decay chains for single decay chains.



Allanach, B.C. et al, Gjelsten, B. K. et al, Miller, D.J. et al, ...

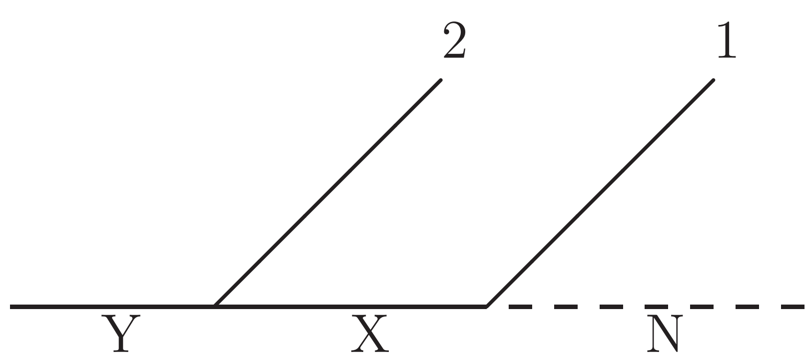


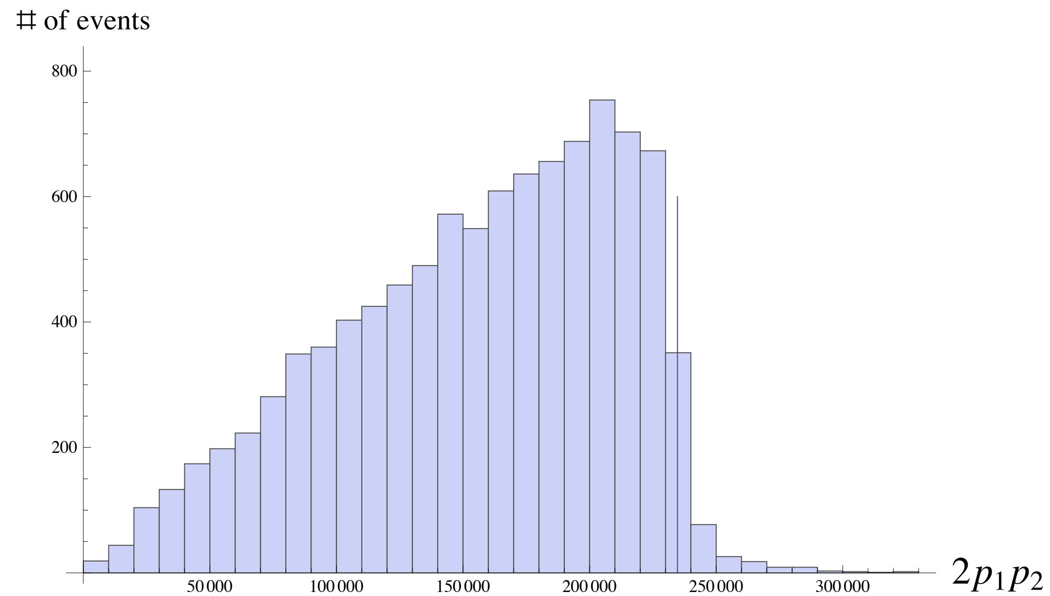
Kawagoe, K. et al, ...

- It's a subprocess of many more complicated events.

Invariant mass end point

- 2 visible particles, only one invariant mass can be formed.



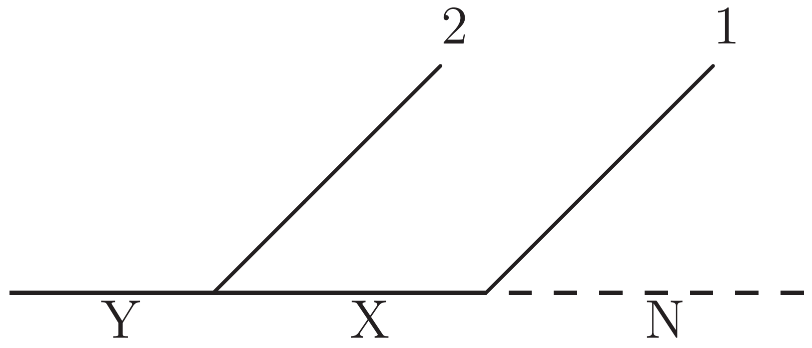


The end point provides one relation among 3 unknown masses, but cannot determine 3 masses individually.

$$(p_1+p_2)^2\Big|_{\max} = \frac{(m_Y^2-m_X^2)(m_X^2-m_N^2)}{m_X^2} \equiv \frac{\Delta_1\Delta_2}{m_X^2}$$

Constraint equations

$$\begin{aligned} p_Y^2 &= m_Y^2, \\ (p_Y - p_2)^2 &= m_X^2, \\ (p_Y - p_2 - p_1)^2 &= m_N^2, \end{aligned}$$



Taking the differences, we get 2 linear equations,

$$\begin{aligned} 2p_2\, p_Y &= m_Y^2 - m_X^2 \equiv \Delta_2, \\ 2p_1\, p_Y - 2p_1\, p_2 &= m_X^2 - m_N^2 \equiv \Delta_1, \end{aligned}$$

assuming $p_1^2 = p_2^2 = 0.$

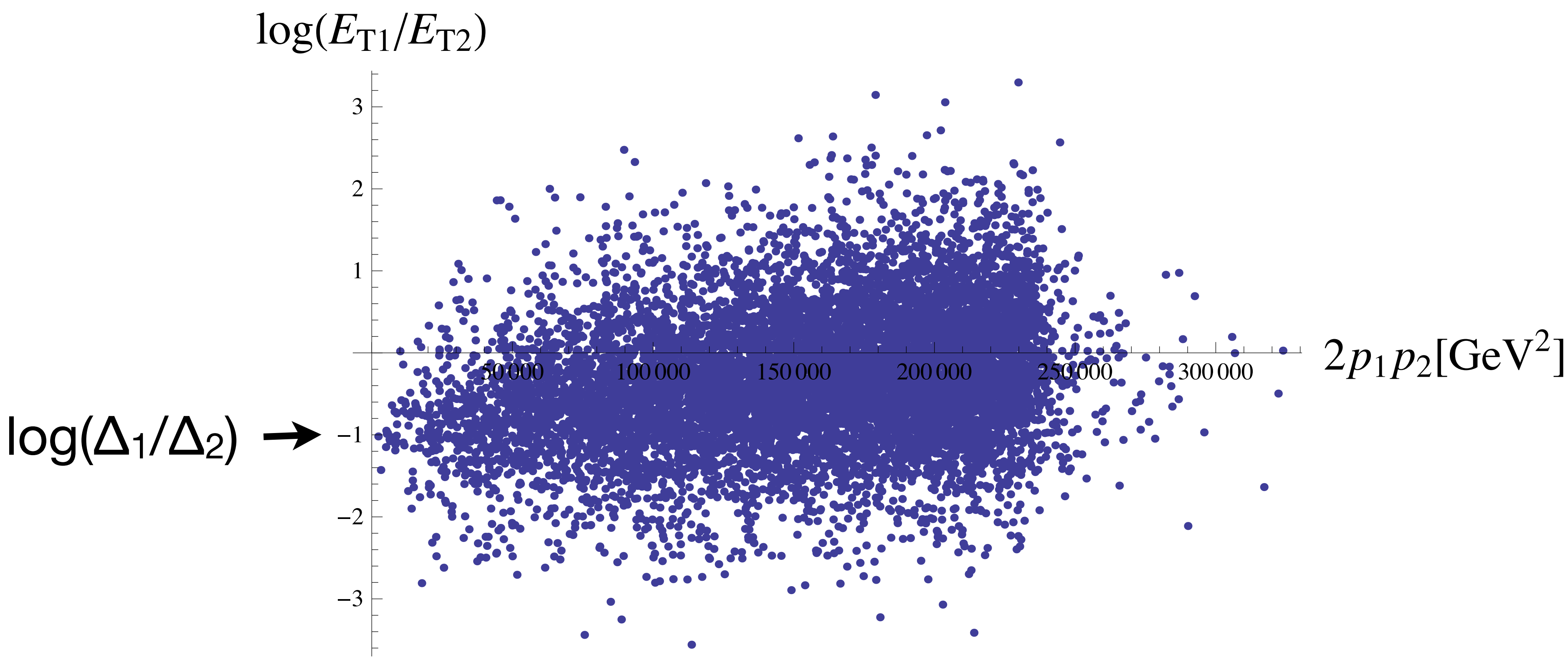
There is another type of special events: when

$$p_1 \parallel p_2 \ \ (p_1 \cdot p_2 = 0) \quad \Rightarrow \quad \frac{E_1}{E_2} = \frac{\Delta_1}{\Delta_2}.$$

We get another relation among 3 masses.

New kinematic variable

- Event distribution in $\log(E_{T1}/E_{T2})$ vs. $2p_1.p_2$ space



	Y	X	N	2	1
particle	left-handed down squark	2nd chargino	anti-sneutrino	up quark	electron
mass[GeV]	777	465	292	0	0

(LM2 point)

New kinematic variable

- Taking the ratio of 2 linear equations, we derive

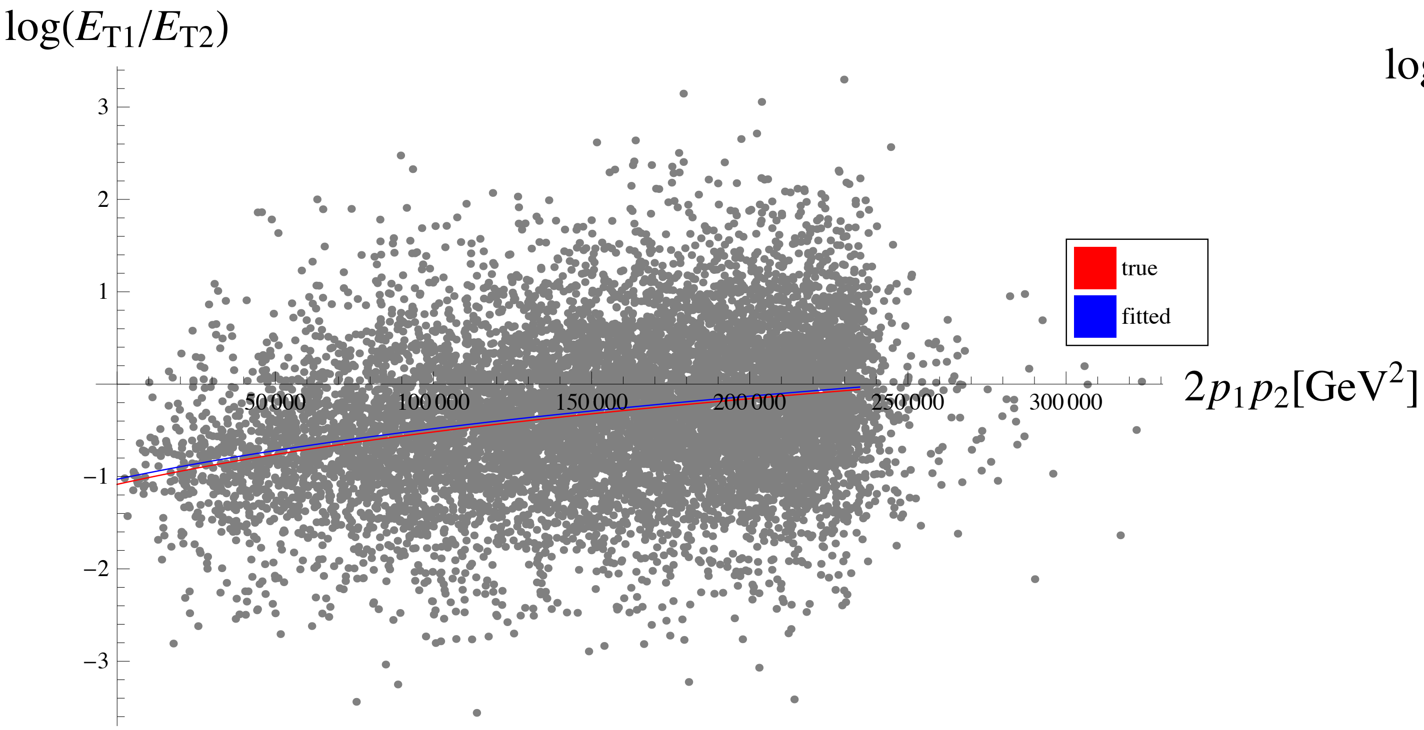
$$\log \frac{E_1}{E_2} = \log \frac{\Delta_1 + 2p_1p_2}{\Delta_2} + \log \left. \frac{1-\beta_Y \cos\theta_{2Y}}{1-\beta_Y \cos\theta_{1Y}} \right|_{\text{lab}}$$
$$= \log \frac{\Delta_1 + 2p_1p_2}{\Delta_2} + \log \left. \frac{1+\beta_Y \cos\theta_{1Y}}{1+\beta_Y \cos\theta_{2Y}} \right|_{Y},$$

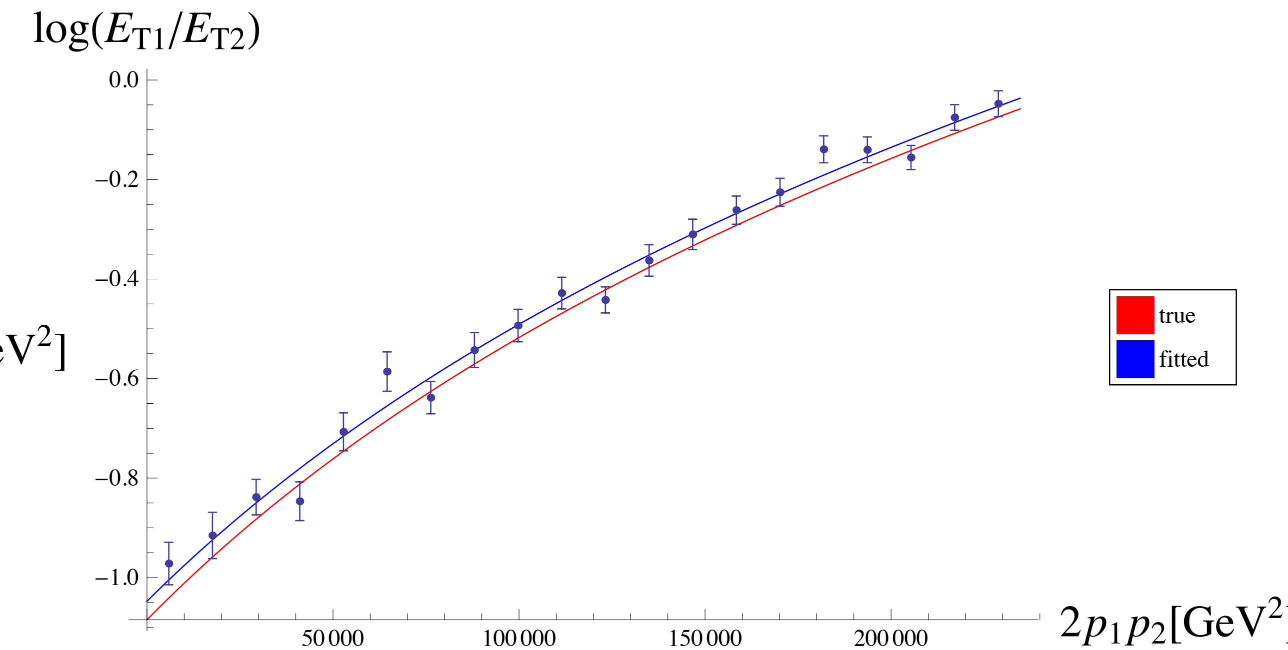
If the 2nd term is randomly distributed around 0, then we can fit the distribution with a curve

$$\log \frac{\Delta_1 + 2p_1p_2}{\Delta_2}$$

to extract Δ_1 and Δ_2 individually. Plus the invariant mass end point, we can solve for 3 masses.

New kinematic variable





	$\Delta_1[\mathrm{GeV}^2]$	$\Delta_2[\mathrm{GeV}^2]$	$\log(\Delta_1/\Delta_2)$	$m_Y[\mathrm{GeV}]$	$m_X[\mathrm{GeV}]$	$m_N[\mathrm{GeV}]$
true	1.310×10^5	3.875×10^5	-1.08	777	465	292
reconstructed	1.370×10^5	3.838×10^5	-1.03	780	473	295
error	$+4.6\%$	-0.96%	$+5.5\%$	$+0.34\%$	$+1.8\%$	$+1.0\%$

for 10^4 parton-level events

It works well for a wide varieties of models and spectra at the parton level.

Comments

- When the mother particle Y is polarized, it creates a bias.
- If Δ_2 (or Δ_1) too small, the p_T cut may induce a fake polarization. The width effect may also cause a bias.
- Many issues in a real experiment:
  - Selecting signals from backgrounds.
  - Experimental smearing.
  - Combinatorial problems.

Combinatorial problems

- Between the 2 visible particles on the decay chain:

  Only $|\log(E_{T1}/E_{T2})|$ can be measured. The distribution is folded along the $\log(E_{T1}/E_{T2})=0$ axis. It causes difficulties if the center of the distribution is close to that axis.

- Between a visible particle on the decay chain and another particle somewhere else:

  ISR jet affects the distribution at small invariant masses. Particles from the other decay chain affect large invariant mass region. We can still fit the middle region to estimate Δ_1 and Δ_2.

Conclusions

- Many kinematic techniques and variables have been developed for new physics signals with missing energies. They are useful for both searches and mass measurements.
- Now there are uniform ways to understand the physical meaning of these variables and to search for new variables and techniques.
- Once the masses of new particles are measured, they can be used to reconstruct the kinematics of the new physics events, which will allow more detailed measurements of other properties such as spins and couplings.